

COLLEGE OF ENGINEERING ROORKEE, ROORKEE
Autumn Semester 2014-2015

Subject: Physics
Theory of Relativity

Subject Code: TPH101
Tutorial Sheet: 2

Numerical Problems

1. An inertial frame S' is moving with velocity $0.6c$ with respect to the frame S along the x -axis. Initially at $t=0$, the origins of the two frames coincide. Two events are recorded in S , one at origin at $t_1=0$ and the other at $x_2=300\text{m}$ and $t_2=10^{-6}\text{sec}$. Find the time interval between the occurrence of the events and separation of positions of their occurrence as measured in S' .
Ans. $5 \times 10^{-7}\text{sec}$, 150m
2. The velocity of a particle is $5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}$ m/sec in a frame of reference moving with uniform velocity $0.5c$ with respect to the laboratory frame along $+x$ -direction. Find the velocity of the particle in the laboratory frame.
Ans. $0.5c\mathbf{i} + 8.6\mathbf{j} + 17.3\mathbf{k}$ m/sec
3. A train whose length is 159m when at rest, has to pass through a tunnel of length 125m . The train is moving with uniform speed of $2.4 \times 10^8\text{m/s}$ towards the tunnel. Find the length of the train and that of tunnel as observed by an observer (i) at the tunnel and (ii) at the train. Ans (i) 90m , 125m (ii) 75m , 150m
4. Show that the circle, $x^2 + y^2 = a^2$ in a frame S appears to be an ellipse in frame S' which is moving with velocity v relative to S .
5. Show that if a system S' frame we have $u'_x = c \cos \phi$ and $u'_y = c \sin \phi$, then in frame S , $u_x^2 + u_y^2 = c^2$ where S' is moving with velocity v with respect to frame S .
6. How fast must an electron move in order that its mass equals the rest mass of the proton.
Ans. $2.99 \times 10^8\text{m/sec}$
7. At what speed should a clock be moved so that it may appear to lose 1 minute in each hour?
Ans. $5.45 \times 10^7\text{m/s}$
8. The half life of a particular particle measured to be $4 \times 10^{-8}\text{second}$ at a speed of $0.8c$. What will be its speed if its half life be $3 \times 10^{-8}\text{second}$? Also find its proper half life.
Ans. $0.6c$, $2.4 \times 10^{-8}\text{sec}$.
9. What is the length of a meter of a meter stick moving parallel to its length when its mass is $3/2$ times of its rest mass?
Ans. 0.667m
10. Show that it is impossible to have photo-electric effect with a free electron.
11. Calculate the percentage contraction of a rod moving with a velocity $0.8c$ in a direction inclined at 30° to its length.
Ans. 27.89%
12. The density of gold is $19.3 \times 10^3\text{kg/m}^3$ in a frame S which is at rest. Calculate its density that an observer in S' would determine if S' is moving along x -axis with a speed $0.9c$.
Ans. $1.016 \times 10^5\text{kg/m}^3$
13. An electron has energy 1GeV . Find the ratio of its mass and its rest mass. Also calculate its velocity.
Ans. 1960 , c
14. Compute the energy released on the conversion of 1gm of matter into energy in kWh.
Ans. $2.5 \times 10^7\text{kWh}$
15. An electron is moving with a speed $0.9c$. Calculate its total energy. Also find the ratio of its non relativistic and relativistic kinetic energy.
Ans. 1.17MeV , 0.31
16. Calculate the speed and the momentum of an electron of kinetic energy 1.02MeV .
Ans. $2.83 \times 10^8\text{m/sec}$, $7.73 \times 10^{-22}\text{kgm/sec}$
17. Proper mean life time of μ -meson is $2 \times 10^{-6}\text{sec}$. These are produced in the atmosphere at a height H and travel towards the earth with a velocity $0.99c$. If only $1/e$ of the mesons initially produced could reach the earth. Find the value of H .
Ans. 4.2km
18. Find the velocity at which the relativistic length of an object differs from its proper length by 2% .
Ans. $5.97 \times 10^7\text{m/s}$
19. Calculate the energy of a photon, the momentum of which is same as that of a proton having kinetic energy 10MeV .
Ans. 137.43MeV

20. Spacecraft A is moving at $0.5c$ with respect to the earth. If another space craft B is to pass A at a relative speed of $0.9c$ in the same direction, what must be the speed of the spacecraft B with respect to the earth. Ans. $0.72c$
21. For what value of β (or v/c) will the relativistic mass of a particle exceed its rest mass by a fraction f ? Ans. $\sqrt{f(2+f)/(1+f)}$
22. At what speed does the kinetic energy of a particle equal its rest energy? If its rest mass is m_0 calculate its mass, momentum and total energy. Ans. $2.6 \times 10^8 \text{ m/s}$, $2m_0$, $2m_0v$, $2m_0c^2$
23. Prove that
 $(1-v^2/c^2)^{-1/2} = \sqrt{(1+p^2c^2/E_0^2)}$
 $(1-v^2/c^2)^{-1/2} = (1+E_K/E_0)$
 $p = \sqrt{[E_K(E_K + 2E_0)]}/c$
 where E_0 and E_K are the rest mass energy and kinetic energy, respectively, p the momentum and v the velocity of particle.
24. A circular lamina moves with its plane parallel to the x - y plane of a reference frame S at rest. Assuming its motion to be along the x -axis calculate the velocity at which its surface area would appear to be reduced to half to an observer in frame S . Also show its shape as seen by him. Ans. $0.866c$
25. An electron is accelerated by a potential of 10^5 keV. Find its speed after the acceleration and compare your result with the speed without taking the relativistic effects.
Ans. $v(\text{non-relativistic})=1.875 \times 10^8 \text{ m/s}$, $v(\text{relativistic})= 1.64 \times 10^8 \text{ m/s}$

Theoretical Questions:

1. State the postulates of special theory of relativity and hence derive the Lorentz transformation equations.
2. Show that Galilean transformation equations are special case of Lorentz transformation equations.
3. Explain the phenomena of length contraction and time dilation. Define proper length and proper time interval. Show that time dilation is real effect.
4. Derive the expression for variation of mass with velocity.
5. Derive Einstein's mass energy relation and give examples to show its validity.
6. Show that velocity of light is the maximum attainable velocity.
7. What are mass-less particles? Find the relation between energy and momentum of these particles.
8. Deduce the formula for relativistic addition of velocities and show that it is consistent with Einstein's II postulate of special theory of relativity.
9. Deduce expression for the relativistic form of Newton's second law.
10. Show that $x^2 + y^2 + z^2 - c^2 t^2$ is invariant under Lorentz transformation.