COLLEGE OF ENGINEERING ROORKEE, ROORKEE
Autumn Semester 2014-2015

Subject: Physics **Subject Code: TPH101**
Wave mechanics **Tutorial Sheet: 3**

Numerical Problems

1. A proton is moving with a speed of $2x10^8$ m/sec. Find the wavelength of the matter wave associated with it Ans. $1.47x\ 10^{-5}$ A
2. Kinetic energy of an electron is $4.55x\ 10^{-25}$ J. Calculate velocity, momentum and wavelength of electron. Ans. 10^3 m/sec, $9.1x\ 10^{-28}$ kg-m/sec and $7.286x\ 10^{-7}$ m
3. Find the de-Broglie wavelength of a neutron of energy 12.8 MeV. Ans. $8.0x\ 10^{-5}$A
4. An electron has a speed of 500 m/s with an accuracy of 0.01%. Calculate the certainty with which its position can be located. Ans. 0.116cm
5. Calculate the smallest possible uncertainty in the position of an electron moving with velocity $3x10^7$ m/sec. Ans. 0.0388A
6. An electron has speed 500 m/s with an accuracy of 0.002%. Calculate the certainty with which we can locate the position of electron. Ans 0.012m
7. An electron is confined to a box of length 10^{-8} m, calculate the minimum uncertainty in its velocity. Ans $1.167x10^4$ m/sec
8. A particle is moving in 1D potential box of infinite height and width 20A. Calculate the probability of finding the particle within an interval 0.2A at the centre of the when it is in the first excited state. Ans. 0.02
9. Find the probability that a particle in a box of width L can be found between x=0 and x=L/n, when it is in the n^{th} state. Ans. 1/n
10. At what speed, the de Broglie wave associated with a particle is equal to twice the Compton wavelength. Ans. $c/\sqrt{5}$
11. Find the frequency of an X-ray photon having momentum $2.2x10^{-23}$kg m/s. Ans. $1x10^{19}$Hz
12. An electron and photon each have a wavelength of 4A. Compare their (a) momentum,(b) total energy,(c) ratio of kinetic energies. Ans (a)$1.657x\ 10^{-24}$kg-ms^{-1},(b) 0.51MeV,3.10keV (c) $3.04x10^{-3}$
13. Calculate the de Broglie wavelength associated with a helium atom at 100^oC. Given that mass of helium atom is $6.7x10^{-27}$kg and the Boltzmann constant $=1.38x10^{-23}$ Joule/Kelvin. Ans. 0.65A
14. A proton in a one dimensional box has energy of 400keV in its first excited state. How wide is the box? Ans. 45.3 fm
15. The phase velocity of ripples on a liquid surface is $[2\pi S/\rho\lambda]^{0.5}$, where S is the surface tension and ρ the density of the liquid. Calculate the group velocity of the ripples. Ans. $3/2\ V_p$
16. The position and momentum of an electron of energy 15KeV are determined simultaneously. The position is found to be located within 0.5A. Calculate the percentage of the uncertainty in the determination of its momentum. Ans. 1.6%

17. If a particle is described by a wave function $\psi(x) = Ax^2$ for $0 < x < 1$, then calculate the probability of finding a particle within x=0.8 and x=0.9. Ans.0.2638
18. A particle is confined to a one dimensional potential box of width 0.2A. It is found that where the particle has energy 230eV, its wave function has 5 nodes. Find the mass of the particle and also Show that it can never have the energy equal to 1KeV. Ans. 5.97×10^{-29}kg

19. Show that if the total energy of a moving particle greatly exceeds its rest energy, its de Broglie wavelength is nearly same as the wavelength of photon with the same total energy.

20. Which of the following wave functions cannot be solutions of Schrodinger equation for all values of x and why? (i) $\psi = A \sec x$, (ii) $\psi = A \tan x$, (iii) $\psi = A \exp(x^2)$

21. The wave function of certain particle is $\psi = A \cos^2 x$ for $-\pi/2 < x < \pi/2$. (a) find the value of A (b) find the probability that the particle be found between x=0 and $x=\pi/4$. Ans. $\sqrt{(8/3\pi)}$, 0.462

Theoretical Questions

1. What is meant by matter waves? Give experimental evidence in support of the existence of these waves.

2. What is a wave packet? Prove that it moves with the group velocity $v_g=v$, the particle velocity.

3. Prove that the group velocity $v_g=v_p - dv_p/d\lambda$, where v_p is the phase velocity of the wave. Also show that $v_p= v_g/2$ for a non relativistic particle.

4. Derive time independent Schrodinger wave equation. What is the significance of a wave function? Also give the conditions for an acceptable wave function.

5. Applying uncertainty principle to show that the presence of protons in the nucleus is possible.

6. Applying the uncertainty principle, to find the minimum energy of a harmonic oscillator.

7. Solve Schrodinger wave equation for a free particle of mass m in an infinite one dimensional potential well of length L and determine the wave functions and energies of the permitted energy levels.

8. Derive Schrodinger time independent wave equation.

9. If m_o is the rest mass of a particle and λ is the wavelength associated with it, then show that its phase velocity is given by $v_p = c\sqrt{[1+(m_o c\lambda/h)^2]}$, c being the velocity of light.
10. Show that the de Broglie wave length associated with a particle of rest mass m_o and relativistic kinetic energy E_k is given by $\lambda = \dfrac{hc}{\sqrt{E_k(E_k + 2m_o c^2)}}$ and if it is a charge particle having charge q and is accelerated through a potential of V volts then its de Broglie wavelength is given by $\lambda = \dfrac{h}{\sqrt{2m_o qV\left(1 + \dfrac{qV}{2m_o c^2}\right)}}$