Discrete Structure Questions & Answers

Discrete Structure : (CO1)

1. Which of the following statement is a proposition?
a) Get me a glass of milkshake
b) God bless you!
c) What is the time now?
d) The only odd prime number is 2
Answer: d
Clarification:- Only this statement has got the truth value which is false.

2. The truth value of '4+3=7 or 5 is not prime'.
a) False
b) True
Answer: b
Clarification:- Compound statement with 'or' is true when either of the statement is true. Here the first part of the statement is true, hence the whole is true.

3. Let P: I am in Bangalore.; Q: I love cricket.; then q -> p(q implies p) is?
a) If I love cricket then I am in Bangalore
b) If I am in Bangalore then I love cricket
c) I am not in Bangalore
d) I love cricket
Answer: a
Clarification:- Q is hypothesis and P is conclusion. So the compound statement will be if hypothesis then conclusion.

4. Let P: I am in Delhi.; Q: Delhi is clean.; then q ^ p(q and p) is?
a) Delhi is clean and I am in Delhi
b) Delhi is not clean or I am in Delhi
c) I am in Delhi and Delhi is not clean
d) Delhi is clean but I am in Mumbai
Answer: a
Clarification:- Connector should be 'and', that is q and p.

5. Let P: This is a great website, Q: You should not come back here. Then 'This is a great website and you should come back here.' is best represented by?
a) ~P V ~Q
b) P ∧ ~Q
c) P V Q
d) P ∧ Q
Answer: b
Clarification:- The second part of the statement is negated, hence negation operator is used.

6. Let P: We should be honest., Q: We should be dedicated., R: We should be overconfident. Then 'We should be honest or dedicated but not overconfident.' is best represented by?

a) ~P V ~Q V R

b) P ∧ ~Q ∧ R

c) P V Q ∧ R

d) P V Q ∧ ~R

Answer: d

Clarification:- The third part of the statement is negated, hence negation operator is used, for ('or' -V) is used and for('but'- ∧).

7. Let P and Q be statements, then P<->Q is logically equivalent to _____

a) P<->~Q

b) ~P<->Q

c) ~P<->~Q

d) None of the mentioned

Answer: c

Clarification:- Both of them have same truth table, Hence they are equal.

8. The compound statement A-> (A->B) is false, then the truth values of A, B are respectively _____

a) T, T

b) F, T

c) T, F

d) F, F

Answer: c

Clarification:- For implications to be false hypothesis should be true and conclusion should be false.

9. The statement which is logically equivalent to A∧ (and) B is?

a) A->B

b) ~A ∧ ~ B

c) A ∧ ~B

d) ~(A->~B)

Answer: d

Clarification:- The truth table of both statements are same.

10."Match will be played only if it is not a humid day." The negation of this statement is?

a) Match will be played but it is a humid day

b) Match will be played or it is a humid day

c) All of the mentioned statement are correct

d) None of the mentioned

Answer: a

Clarification:- Negation of P->Q is P∧~Q.

11.Consider the following statements.

A: Raju should exercise.

B: Raju is not a decent table tennis player.

C: Raju wants to play good table tennis.

The symbolic form of "Raju is not a decent table tennis player and if he wants to play good table tennis then he should exercise." is?

a) A->B->C

b) B∧(C->A)

c) C->B∧A

d) B<->A∧C

Answer: b

Clarification:- For conditionals statement (if then), implications are used.

12.The statement (~P<->Q) ∧ ~Q is true when?

a) P: True Q: False

b) P: True Q: True

c) P: False Q: True

d) P: False Q: False

Answer: a

Clarification:- For a bi-conditional to be true both inputs should be same.

13.Let P, Q, R be true, false, false, respectively, which of the following is true?

a) P∧(Q∧~R)

b) (P->Q)∧~R

c) Q<->(P∧R)

d) P<->(QvR)

Answer: c

Clarification:- For a bi-conditional to be true both inputs should be the same.

14.Which of the following statements is the negation of the statements "4 is odd or -9 is positive"?

a) 4 is even or −9 is not negative

b) 4 is odd or −9 is not negative

c) 4 is even and −9 is negative

d) 4 is odd and −9 is not negative

Answer: c

Clarification:- Using De Morgan's Law ~(A V B) ↔ ~A ∧ ~B.

15.Which of the following represents: ~A (negation of A) if A stands for "I like badminton but hate maths"?

a) I hate badminton and maths

b) I do not like badminton or maths

c) I dislike badminton but love maths

d) I hate badminton or like maths

Answer: d

Clarification:- De Morgan's Law ~ (A ∧ B) ↔ ~A V ~B.

Discrete Structure : (CO2)

1.A compound proposition that is always_____is called a tautology.

a) True

b) False

Answer: a

Clarification:- Tautology is always true.

2.A compound proposition that is always_____is called a contradiction.

a) True

b) False

Answer: b

Clarification:- Contradiction is always false.

3.If A is any statement, then which of the following is a tautology?

a) A ∧ F

b) A ∨ F

c) A ∨ ¬A

d) A ∧ T

Answer: c

Clarification:- A ∨ ¬A is always true.

4.If A is any statement, then which of the following is not a contradiction?

a) A ∧ ¬A

b) A ∨ F

c) A ∧ F

d) None of mentioned

Answer: b

Clarification:- A ∨ F is not always false.

5.A compound proposition that is neither a tautology nor a contradiction is called a _____

a) Contingency

b) Equivalence

c) Condition

d) Inference

Answer: a

Clarification:- Definition of contingency.

6.p → q is logically equivalent to _____

a) ¬p ∨ ¬q

b) p ∨ ¬q

c) ¬p ∨ q

d) ¬p ∧ q

Answer: c

Clarification:- (p → q) ↔ (¬p ∨ q) is tautology.

7.p ∨ q is logically equivalent to _____

a) ¬q → ¬p

b) q → p

c) ¬p → ¬q

d) ¬p → q

Answer: d

Clarification:- (p ∨ q) ↔ (¬p → q) is tautology.

8.¬ (p ↔ q) is logically equivalent to _____

a) q↔p

b) p↔¬q

c) ¬p↔¬q

d) ¬q↔¬p

Answer: b

Clarification:- ¬(p↔q)↔(p↔¬q) is tautology.

9.Which of the following statement is correct?

a) $p \vee q \equiv q \vee p$

b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

c) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

d) All of mentioned

Answer: d

Clarification:- Verify using truth table, all are correct.

10.$(p \rightarrow q) \wedge (p \rightarrow r)$ is logically equivalent to _____

a) $p \rightarrow (q \wedge r)$

b) $p \rightarrow (q \vee r)$

c) $p \wedge (q \vee r)$

d) $p \vee (q \wedge r)$

Answer: a

Clarification:- $((p \rightarrow q) \wedge (p \rightarrow r)) \leftrightarrow (p \rightarrow (q \wedge r))$ is tautology.

Discrete Structure : (CO3)

1.A non empty set A is termed as an algebraic structure _____

a) with respect to binary operation *

b) with respect to ternary operation ?

c) with respect to binary operation +

d) with respect to unary operation -

Answer: a

Clarification:- A non empty set A is called an algebraic structure w.r.t binary operation "*" if (a*b) belongs to S for all (a*b) belongs to S. Therefore "*" is closure operation on 'A'.

2. An infinite cyclic group does not have a_____series.

a) AP

b) GP

c) Composite

d) Finite

Answer: c

Clarification:- Suppose that any finite group of order less than n has a composition series. Let G be a finite group of order n. If G is simple, then $G \triangleright \{e\}$, where e is the identity element of G and hence, it is a composition series. However, any infinite cyclic group does not have a composite series.

3. Every cyclic group is a/an _____

a) infinite subgroup

b) abelian group

c) monoid

d) commutative semigroup

Answer: b

Clarification:- Let C be a cyclic group with a generator $g \in C$. Namely, we have G={g.Let x and y be arbitrary elements in C. Then, there exists n, $m \in Z$ such that x=gn and y=gm. It follows that x*y = gn*gm = gn+m = gm*gn = yx. Hence, we find that xy=yx for any x,y belongs to G.Thus, G is an abelian group.

4. What is an irreducible module?

a) A cyclic module in a ring with any non-zero element as its generator

b) A cyclic module in a ring with any positive integer as its generator

c) An acyclic module in a ring with rational elements as its generator

d) A linearly independent module in a semigroup with a set of real numbers

Answer: a

Clarification:- A nonzero R-module M is irreducible if and only if M is a cyclic module with any nonzero element as its generator. Suppose that M is an irreducible module. Let $a \in M$ be any nonzero element and consider the submodule (a) generated by the element a. Since a is a nonzero element, the submodule (a) is non-zero. Since M is irreducible, this implies that M=(a). Hence M is a cyclic module generated by a. Since a is any nonzero element, the module M is a cyclic module with any nonzero element as its generator.

5. A finite group G of order 219 is _____

a) a semigroup
b) a subgroup
c) a commutative inverse
d) a cyclic group

Answer: d

Clarification:- The prime factorization $219=3 \cdot 73$. By the definition of Sylow's theorem, determine the number np of Sylow p-group for p=3,73. $np \equiv 1 \pmod p$ and np divides n/p. Thus, n3 could be 1, 4, 7, 10, 13,... and n3 needs to divide 219/3=73. Hence the only possible value for n3 is n3=1. So there is a unique Sylow 3-subgroup P3 of G. By Sylow's theorem, the unique Sylow 3-subgroup must be a normal subgroup of G. Similarly, n73=1, 74,... and n73 must divide 219/73=3 and hence we must have n73=1. Thus, G has a unique normal Sylow 73-subgroup P73.

6. The number of generators of cyclic group of order 219 is _____

a) 144
b) 124
c) 56
d) 218

Answer: a

Clarification:- The number of generators of a cyclic group of order n is equal to the number of integers between 1 and n that are relatively prime to n.Namely, the number of generators is equal to $\varphi(n)$, where φ is the Euler totient function. We know that G is a cyclic group of order 219. Hence, the number of generators of G is $\varphi(219) = \varphi(3)\varphi(73) = 3 \cdot 73 = 144$.

7. The order of a simple abelian group is _____

a) infinite
b) real number
c) finite
d) prime

Answer: a

Clarification:- Let p be the order of g (hence the order of G). As a contradiction, assume that p=ab is a composite number with integers $a > 1$, $b > 1$. Then (ga) is a proper normal subgroup of G. This is a contradiction since G is simple. Thus, p must be a prime number.
Therefore, the order of G is a prime number.

8. The Number of Elements Satisfying $g7=e$ in a finite Group F is _____

a) even
b) not a number
c) odd
d) rational

Answer: c

Clarification:- Let $g \neq e$ be an element in group F such that $g7=e$. As 7 is a prime number, this yields that the order of g is 7. Consider, the subgroup (g) is generated by g. As the order of g is 7, the order of the subgroup (g) is 7. Hence, the order must be odd.

9. All the rings of order p2 is _____

a) associative

b) cyclic

c) inverse

d) commutative

Answer: d

Clarification:- Let R be a ring with unit 1. Suppose that the order of R is |R|=p2 for some prime number p. Then it has been proven that R is a commutative ring.

10.A group G of order 20 is _____

a) solvable

b) unsolvable

c) 1

d) not determined

Answer: a

Clarification:- The prime factorization of 20 is 20=2·5. Let n5 be the number of 5-Sylow subgroups of G. By Sylow's theorem, we have, n5≡1(mod 5) and n5|4. Thus, we have n5=1. Let P be the unique 5-Sylow subgroup of G. The subgroup P is normal in G as it is the unique 5-Sylow subgroup. Then consider the subnormal series G▹P▹{e}, where e is the identity element of G. Then the factor groups G/P, P/{e} have order 4 and 5 respectively. Hence these are cyclic groups (in particular abelian). Hence, the group G of order 20 has a subnormal series whose factor groups are abelian groups, and thus G is a solvable group.

Discrete Structure : (CO4)

1.A_____is an ordered collection of objects.

a) Relation
b) Function
c) Set
d) Proposition

Answer: c

Clarification:- By the definition of set.

2.Power set of empty set has exactly_____subset.

a) One
b) Two
c) Zero
d) Three

Answer: a

Clarification:- Power set of null set has exactly one subset which is empty set.

3.What is the Cartesian product of A = {1, 2} and B = {a, b}?

a) {(1, a), (1, b), (2, a), (b, b)}
b) {(1, 1), (2, 2), (a, a), (b, b)}
c) {(1, a), (2, a), (1, b), (2, b)}
d) {(1, 1), (a, a), (2, a), (1, b)}

Answer: c

Clarification:- A subset R of the Cartesian product A x B is a relation from the set A to the set B.

4.The Cartesian Product B x A is equal to the Cartesian product A x B.

a) True
b) False

Answer: b

Clarification:- Let A = {1, 2} and B = {a, b}. The Cartesian product A x B = {(1, a), (1, b), (2, a), (2, b)} and the Cartesian product B x A = {(a, 1), (a, 2), (b, 1), (b, 2)}. This is not equal to A x B.

5.Which of the following two sets are equal?

a) A = {1, 2} and B = {1}
b) A = {1, 2} and B = {1, 2, 3}
c) A = {1, 2, 3} and B = {2, 1, 3}
d) A = {1, 2, 4} and B = {1, 2, 3}

Answer: c

Clarification:- Two set are equal if and only if they have the same elements.

6.The union of the sets {1, 2, 5} and {1, 2, 6} is the set _____

a) {1, 2, 6, 1}
b) {1, 2, 5, 6}
c) {1, 2, 1, 2}
d) {1, 5, 6, 3}

Answer: b

Clarification:- The union of the sets A and B, is the set that contains those elements that are either in A or in B.

7.The intersection of the sets {1, 2, 5} and {1, 2, 6} is the set _____

a) {1, 2}
b) {5, 6}
c) {2, 5}
d) {1, 6}

Answer: a

Clarification:- The intersection of the sets A and B, is the set containing those elements that are in both A and B.

8. **Two sets are called disjoint if there_____is the empty set.**

a) Union

b) Difference

c) Intersection

d) Complement

Answer: c

Clarification:- By the definition of the disjoint set.

9. **Which of the following two sets are disjoint?**

a) {1, 3, 5} and {1, 3, 6}

b) {1, 2, 3} and {1, 2, 3}

c) {1, 3, 5} and {2, 3, 4}

d) {1, 3, 5} and {2, 4, 6}

Answer: d

Clarification:- Two sets are disjoint if the intersection of two sets is the empty set.

10. **The difference of {1, 2, 3} and {1, 2, 5} is the set _____**

a) {1}

b) {5}

c) {3}

d) {2}

Answer: c

Clarification:- The difference of the sets A and B denoted by A-B, is the set containing those elements that are in A not in B.

11. **The complement of the set A is _____**

a) A – B

b) U – A

c) A – U

d) B – A

Answer: b

Clarification:- The complement of the set A is the complement of A with respect to U.

12. **The set difference of the set A with null set is _____**

a) A

b) Null

c) U

d) B

Answer: a

Clarification:- The set difference of the set A by the null set denoted by A – {null} is A.

13. **Let the set A is {1, 2, 3} and B is {2, 3, 4}. Then the set A – B is?**

a) {1, -4}

b) {1, 2, 3}

c) {1}

d) {2, 3}

Answer: c

Clarification:- In A – B the common elements get cancelled.

14. In which of the following sets A – B is equal to B – A?

a) A = {1, 2, 3}, B = {2, 3, 4}

b) A = {1, 2, 3}, B = {1, 2, 3, 4}

c) A = {1, 2, 3}, B = {2, 3, 1}

d) A = {1, 2, 3, 4, 5, 6}, B = {2, 3, 4, 5, 1}

Answer: c

Clarification:- A- B= B-A = Empty set.

15. If A is {{Φ}, {Φ, {Φ}}}, then the power set of A has how many element?

a) 2

b) 4

c) 6

d) 8

Answer: b

Clarification:- The set A has got 2 elements so n(P(A))=4.

16. Two sets A and B contains a and b elements respectively. If power set of A contains 16 more elements than that of B, value of 'b' and 'a' are _____

a) 4, 5

b) 6, 7

c) 2, 3

d) None of the mentioned

Answer: a

Clarification:- 32-16=16, hence a=5, b=4.

17. Let A be {1, 2, 3, 4}, U be set of all natural numbers, then U-A'(complement of A) is given by set.

a) {1, 2, 3, 4, 5, 6, ….}

b) {5, 6, 7, 8, 9, ……}

c) {1, 2, 3, 4}

d) All of the mentioned

Answer: c

Clarification:- U – A' ≡ A.

18.A function is said to be_____if and only if f(a) = f(b) implies that a = b for all a and b in the domain of f.

a) One-to-many

b) One-to-one

c) Many-to-many

d) Many-to-one

Answer: b

Clarification:- A function is one-to-one if and only if f(a)≠f(b) whenever a≠b.

19.The inverse of function f(x) = x3 + 2 is _____

a) f -1 (y) = (y – 2) 1/2

b) f -1 (y) = (y – 2) 1/3

c) f -1 (y) = (y) 1/3

d) f -1 (y) = (y – 2)

Answer: b

Clarification:- To find the inverse of the function equate f(x) then find the value of x in terms of y such that f -1 (y) = x.

20.A Poset in which every pair of elements has both a least upper bound and a greatest lower bound is termed as _____

a) sublattice

b) lattice

c) trail

d) walk

Answer: b

Clarification:- A poset in which every pair of elements has both a least upper bound and a greatest lower bound is called a lattice. A lattice can contain sublattices which are subsets of that lattice.

Discrete Structure : (CO5)

1.A non empty set A is termed as an algebraic structure _____

e) with respect to binary operation *
f) with respect to ternary operation ?
g) with respect to binary operation +
h) with respect to unary operation -

Answer: a

Clarification:- A non empty set A is called an algebraic structure w.r.t binary operation "*" if (a*b) belongs to S for all (a*b) belongs to S. Therefore "*" is closure operation on 'A'.

2.An algebraic structure_____is called a semigroup.

a) (P, *)
b) (Q, +, *)
c) (P, +)
d) (+, *)

Answer: a

Clarification:- An algebraic structure (P,*) is called a semigroup if a*(b*c) = (a*b)*c for all a,b,c belongs to S or the elements follow associative property under "*". (Matrix,*) and (Set of integers,+) are examples of semigroup.

3.Condition for monoid is _____

a) (a+e)=a
b) (a*e)=(a+e)
c) a=(a*(a+e)
d) (a*e)=(e*a)=a

Answer: d

Clarification:- A Semigroup (S,*) is defined as a monoid if there exists an element e in S such that (a*e) = (e*a) = a for all a in S. This element is called identity element of S w.r.t *.

4.A monoid is called a group if _____

a) (a*a)=a=(a+c)
b) (a*c)=(a+c)
c) (a+c)=a
d) (a*c)=(c*a)=e

Answer: d

Clarification:- A monoid(B,*) is called Group if to each element there exists an element c such that (a*c)=(c*a)=e. Here e is called an identity element and c is defined as the inverse of the corresponding element.

5.A group (M,*) is said to be abelian if _____

a) (x+y)=(y+x)
b) (x*y)=(y*x)
c) (x+y)=x
d) (y*x)=(x+y)

Answer: b

Clarification:- A group (M,*) is said to be abelian if (x*y) = (x*y) for all x, y belongs to M. Thus Commutative property should hold in a group.

6.Matrix multiplication is a/an_____property.

a) Commutative
b) Associative
c) Additive
d) Disjunctive

Answer: b

Clarification:- The set of two M*M non-singular matrices form a group under matrix multiplication operation. Since matrix multiplication is itself associative, it holds associative property.

7.A cyclic group can be generated by a/an_____element.

a) singular
b) non-singular
c) inverse
d) multiplicative

Answer: a

Clarification:- A singular element can generate a cyclic group. Every element of a cyclic group is a power of some specific element which is known as a generator 'g'.

8.How many properties can be held by a group?

a) 2
b) 3
c) 5
d) 4

Answer: c

Clarification:- A group holds five properties simultaneously -

i) Closure
ii) associative
iii) Commutative
iv) Identity element
v) Inverse element.

9.A cyclic group is always _____

a) abelian group
b) monoid
c) semigroup
d) subgroup

Answer: a

Clarification:- A cyclic group is always an abelian group but every abelian group is not a cyclic group. For instance, the rational numbers under addition is an abelian group but is not a cyclic one.

10.{1, i, -i, -1} is _____

a) semigroup
b) subgroup
c) cyclic group
d) abelian group

Answer: c

Clarification:- The set of complex numbers {1, i, −i, −1} under multiplication operation is a cyclic group. Two generators i and −i will covers all the elements of this group. Hence, it is a cyclic group.

11.A trivial subgroup consists of _____

a) Identity element
b) Coset
c) Inverse element
d) Ring

Answer: a

Clarification:- Let G be a group under a binary operation * and a subset H of G is called a subgroup of G if H forms a group under the operation *. The trivial subgroup of any group is the subgroup consisting of only the Identity element.

12. **Minimum subgroup of a group is called _____**

a) a commutative subgroup

b) a lattice

c) a trivial group

d) a monoid

Answer: c

Clarification:- The subgroups of any given group form a complete lattice under inclusion termed as a lattice of subgroups. If o is the Identity element of a group(G), then the trivial group(o) is the minimum subgroup of that group and G is the maximum subgroup.

13. **Let K be a group with 8 elements. Let H be a subgroup of K and H<K. It is known that the size of H is at least 3. The size of H is _____**

a) 8

b) 2

c) 3

d) 4

Answer: d

Clarification:- For any finite group G, the order (number of elements) of every subgroup L of G divides the order of G. G has 8 elements. Factors of 8 are 1, 2, 4 and 8. Since given the size of L is at least 3(1 and 2 eliminated) and not equal to G(8 eliminated), the only size left is 4. Size of L is 4.

14. **_____is not necessarily a property of a Group.**

a) Commutativity

b) Existence of inverse for every element

c) Existence of Identity

d) Associativity

Answer: a

Clarification:- Grupoid has closure property; semigroup has closure and associative; monoid has closure, associative and identity property; group has closure, associative, identity and inverse; the abelian group has group property and commutative.

15. **A group of rational numbers is an example of _____**

a) a subgroup of a group of integers

b) a subgroup of a group of real numbers

c) a subgroup of a group of irrational numbers

d) a subgroup of a group of complex numbers

Answer: b

Clarification:- If we consider the abelian group as a group rational numbers under binary operation + then it is an example of a subgroup of a group of real numbers.

16. **Intersection of subgroups is a _____**

a) group

b) subgroup

c) semigroup

d) cyclic group

Answer: b

Clarification:- The subgroup property is intersection closed. An arbitrary (nonempty) intersection of subgroups with this property, also attains the similar property.

17. The group of matrices with determinant_____is a subgroup of the group of invertible matrices under multiplication.

a) 2
b) 3
c) 1
d) 4

Answer: c

Clarification:- The group of real matrices with determinant 1 is a subgroup of the group of invertible real matrices, both equipped with matrix multiplication. It has to be shown that the product of two matrices with determinant 1 is another matrix with determinant 1, but this is immediate from the multiplicative property of the determinant. This group is usually denoted by(n, R).

18. What is a circle group?

a) a subgroup complex numbers having magnitude 1 of the group of nonzero complex elements
b) a subgroup rational numbers having magnitude 2 of the group of real elements
c) a subgroup irrational numbers having magnitude 2 of the group of nonzero complex elements
d) a subgroup complex numbers having magnitude 1 of the group of whole numbers

Answer: a

Clarification:- The set of complex numbers with magnitude 1 is a subgroup of the nonzero complex numbers associated with multiplication. It is called the circle group as its elements form the unit circle.

19.A normal subgroup is _____

a) a subgroup under multiplication by the elements of the group
b) an invariant under closure by the elements of that group
c) a monoid with same number of elements of the original group
d) an invariant equipped with conjugation by the elements of original group

Answer: d

Clarification:- A normal subgroup is a subgroup that is invariant under conjugation by any element of the original group that is, K is normal if and only if gKg−1=K for any g belongs to G Equivalently, a subgroup K of G is normal if and only if gK=Kg for any g belongs to G.Normal subgroups are useful in constructing quotient groups and in analyzing homomorphisms.

20. Two groups are isomorphic if and only if_____is existed between them.

a) homomorphism
b) endomorphism
c) isomorphism
d) association

Answer: c

Clarification:- Two groups M and K are isomorphic (M ~= K) if and only if there exists an isomorphism between them. An isomorphism f:M -> K between two groups M and K is a mapping which satisfies two conditions: 1) f is a bijection and 2) for every x,y belongs to M, we have f(x*My) = f(x) * Kf(y).